Differential Feedback in Codebook-Based Multiuser MIMO Systems in Slowly Varying Channels

Kyeongyeon Kim, *Member, IEEE,* Taejoon Kim, *Student Member, IEEE,* David J. Love, *Senior Member, IEEE,* and Il Han Kim, *Member, IEEE*

***Abstract*—In downlink multiuser multiple-input multiple-output (MIMO) systems, system performance highly depends on the reliability of downlink channel state information (CSI) at the base station (BS). In frequency division duplexing, the most practical solution is to have downlink CSI from the users fed back to the BS. Most work on this feedback design has assumed independent block fading channels. However, this paper proposes a new differential feedback scheme using the observation that the channel realizations are usually temporally correlated. The sum-rate loss assuming differential feedback is analyzed for a system with the number of users K equal to the number of transmit antennas M. When $K > M$, a user selection algorithm based on an approximated signal-to-interference plus noise ratio (SINR) estimation is proposed. In simulation results, the proposed differential feedback scheme increases the sum-rate compared to previous differential feedback schemes. Moreover, the proposed user selection algorithm outperforms semi-orthogonal user selection in the moderate signal-to-noise ratio (SNR) region, despite requiring less feedback information. In low mobility channels, utilizing the channels' time correlation during quantization is shown to play a bigger role in determining sum-rate performance than multiuser diversity for most SNR regimes when a practical number of users is considered.**

***Index Terms*—Multiuser MIMO, limited feedback, time correlation, differential feedback, user selection, sum-rate.**

I. Introduction

THERE has been substantial recent attention focused on multiuser multiple-input multiple-output (MIMO) systems because of their potential benefits in next generation multiple antenna cellular networks [1]. To handle multiuser interference or leverage multiuser diversity in the downlink, the transmitter requires some form of downlink channel state information (CSI). Often, this CSI is obtained through feedback sent by the users over the uplink channels. There has been a large amount of research over the last few years aimed at limiting the amount of feedback overhead needed for multiuser MIMO systems (e.g., see the references in [2]). Most of this research is based on designing and using CSI codebooks. Prior research on limited feedback schemes (both single user and multiuser) has primarily considered independent and identically distributed (i.i.d.) block fading channels (e.g., see [3]–[7]).

However, temporally correlated channels are observed in deployed wireless systems [8]. There have been multiple investigations into using the channel's time correlation in single-user feedback systems. Subspace tracking based gradient feedback approaches are studied in [9]–[11]. The work in [12] quantizes a geodesic trajectory connecting two subspaces, instead of quantizing the subspaces themselves, to facilitate tracking. For spatially or temporally correlated channels, a codebook switching method is considered in [13]. To reduce the memory required for codebook switching, a systematic codebook design is required such as rotation and scaling of a mother codebook [14], [15]. Systematic codebook techniques can be employed to update the quantization codebook using spatial or temporal channel correlation information [15]. In slowly varying channels, [16] proposes using one bit of feedback per angle parameter in the Givens rotation parametrization of a unitary matrix. Other quantized update techniques for CSI feedback include using extrapolation [17], a rotation based differential codebook [18]–[21], feeding back the complementary subspace of a differential rotation matrix [22], and using a predictive codebook based on steady state performance analysis [23]. Differential feedback approaches that quantize a difference vector or matrix have also been studied for long term covariance matrix feedback [24], MIMO single carrier frequency division multiple access (SC-FDMA) with feedback [25], and CSI feedback in the presence of channel estimation error and quantization distortion [26]. As well, the CSI feedback rate has been analyzed by modeling the CSI quantization as a first-order finite-state Markov chain, and a feedback compression scheme is proposed in [27]. Also the feedback is compressed by using entropy coding in [28] and [29].

There has only been limited work investigating the use of the channels' time correlation properties in multiuser MIMO feedback systems. A codebook update method utilizing time correlation for multiuser systems is proposed in [30]–[32]. The primary idea is to use a predictive quantization method, quantize the difference between the present target vector and

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K. Kim is with the Samsung Advanced Institute of Technology (SAIT), Samsung Electronics, Mt. 14-1, Nongseo-dong, Giheung-gu, Yongin-si Gyeonggi-do, South Korea (e-mail: kyeongyeon.kim@samsung.com).

T. Kim is with the Nokia Research Center (NRC), Berkeley, CA 94704, USA (e-mail: taejoon.1.kim@nokia.com).

D. J. Love is with the Department of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47906, USA (e-mail: djlove@purdue.edu).

I.-H. Kim is with the Systems and Applications R&D Center, Texas Instruments Incorporated, Dallas, TX 75243, USA (e-mail: il-han-kim@ti.com).

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the previously quantized vector, and predict the current target vector from the quantized difference vector and previously quantized vector [33]. The update quantizers in [31] and [32] work with channel direction information (CDI). While a predictive quantizer in [31] is designed in Euclidean space and the quantizer output is normalized, a predictive quantizer in [32] is proposed directly on the Grassmannian manifold.

One major issue with CDI feedback is that it is insufficient to exploit multiuser diversity as shown in [34]. In [30], quantization of the entire unnormalized channel vector is considered, but the predictive quantizer does not take into account the previous quantization errors which can have a significant impact on the quality of the base station's (BS's) CSI. To reduce the effect of previous quantization error, high resolution feedback is suggested at the first feedback period. Recently, [35]–[37] propose a differential feedback of the channel Gram matrix on the geodesic curves for block diagonalized multiuser MIMO, and [38] exploits channel correlation on the Grassmannian manifold by using a progressive refinement codebook based on a base codebook and scaling or rotating a local codebook.

In this paper, we develop a differential feedback scheme utilizing temporal correlation that can be applied to single user MIMO and multiuser MIMO. We first focus on point-to-point channel quantization. A predictive quantization method for the entire channel vector (i.e., not just the channel direction) is considered at each of the users. Unlike [30], the impact of quantization error during the update process is explicitly taken into account. Upper and lower bounds on the quantization distortion are analyzed based on a Gaussian assumption for the CSI quantization error. The lower bound, inspired by rate distortion theory, is used in the proposed differential feedback scheme.

The proposed differential feedback scheme is applied to a multiuser MIMO system using zero-forcing precoding[1]. When the number of users K is equal to the number of transmit antennas M, we analyze the impact of the feedback amount on the sum-rate performance. In addition, multiuser diversity is exploited using the proposed feedback scheme when $K > M$. Most of the previous user selection algorithms for limited feedback multiuser MIMO have considered only CDI during the precoder design [31], [34], [39]–[44]. If additional channel quality information (CQI), such as the channel power or signal-to-interference plus noise ratio (SINR), is available, the scheduler can select users to maximize the sum-rate or some other function of the SINRs [31], [34], [39]–[44]. We estimate SINR at the BS using quantized channel information and our residual quantization distortion bound. We then propose a user selection algorithm using this estimated SINR that does not require any additional user feedback (such as additional channel quality of norm information that is often implicitly assumed).

This paper is organized as follows. Section II presents a multiuser MIMO broadcasting system using zero-forcing precoding and limited feedback. In Section III, we propose a new differential feedback scheme using differential quantization analysis. For a multiuser system with a zero-forcing precoder, the average sum-rate is analyzed when $K = M$, and a user selection algorithm using the proposed feedback scheme is proposed when $K > M$. Section V validates the performance of the proposed feedback scheme and user selection algorithm by comparing with previously published schemes. Finally, Section VI gives concluding remarks.

[1]The proposed differential feedback algorithm can be applied to any kind of multiuser precoder, but the zero-forcing precoder is considered for simplicity.

II. SYSTEM OVERVIEW

Consider a multiuser MIMO system that consists of a BS with M antennas and K users each having a single antenna. Assume that each user's channel is slowly varying, spatially uncorrelated, and flat. When the channel is modeled by a Gauss-Markov process with time correlation parameter $a_k \in \mathbb{R}$, $0 \le a_k \le 1$ given, the channel vector of the kth user at the nth time is expressed by

$$\mathbf{h}_k[n] = a_k \mathbf{h}_k[n-1] + \sqrt{1-a_k^2}\boldsymbol{\delta}_k[n], \tag{1}$$

where $\boldsymbol{\delta}_k[n], \mathbf{h}_k[n-1] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}) \in \mathbb{C}^{M\times 1}$ and $E\left(\mathbf{h}_k[n-1]\boldsymbol{\delta}_k{}^H[n]\right) = \mathbf{O}$, where $\mathbf{O}$ denotes the $M \times M$ all zero matrix. Throughout this paper, we assume that the channels of all K users are spatially i.i.d.

Compared to optimal nonlinear encoding, linear precoding schemes such as zero-forcing or minimum mean square error precoding cannot achieve the sum capacity but have been exploited for their simplicity [7]. In typical systems, the number of users K is greater than the number of transmit antennas M, and user scheduling algorithms have been employed to exploit multiuser diversity [31], [34], [39]–[43], [45], [46]. Although a scheduler maximizing sum-rate is optimal, tractable but suboptimal user selection algorithms such as orthogonal or semi-orthogonal user selection (SUS) are proposed in [34], [41], [42], [45], [46]. In this paper, we consider a scheduler to maximize the sum-rate of M users out of K users and a zero-forcing precoder.

Our system model is given by a scheduler and zero-forcing precoder with finite rate feedback information as shown in Fig. 1. Each time the BS schedules users, feedback is assumed to be received from all K users. Each of the K users selects $\hat{\mathbf{h}}_k[0]$ to quantize $\mathbf{h}_k[0]$ from a B-bit finite set, which is called a *codebook* and is known to both the BS and each of the users, and feeds back the selected codeword index. The BS can utilize only the quantized channel vector given by $\hat{\mathbf{h}}_k[0]$ as the CSI for the kth user for scheduling and multiuser precoder. The scheduler selects M users out of K users to maximize the sum-rate by using an estimated SINR for the $\binom{K}{M}$ possible combined channel matrices, $\hat{\mathbf{H}}[0]$, formed by stacking the selected users' channel vectors as will be discussed in Section IV-B. Throughout a scheduled transmission, the proposed differential CSI feedback is assumed to take place for all scheduled M users.

Let the selected user set be $\mathcal{S}$. In the considered linear precoding system, the jth user's information signal in the selected user set $\mathcal{S}$ is multiplied by its linear precoding vector $\mathbf{v}_j[n]$, and then the transmitted signal is $\mathbf{x}[n] = \sum_{j\in\mathcal{S}} \mathbf{v}_j[n]s_j[n]$, where $E\left(\|\mathbf{x}[n]\|^2\right) \le P$ for total transmit power P and $s_j[n]$ is the data signal for the jth user. The received signal at the kth scheduled mobile is

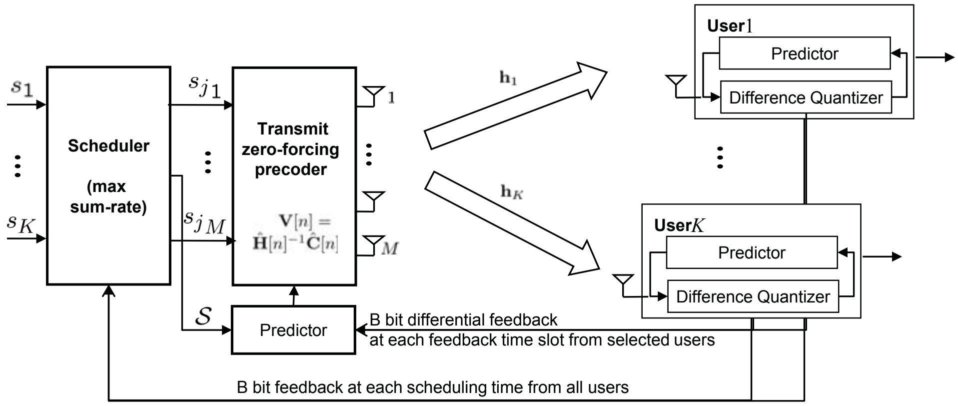

Fig. 1. System model of a scheduler maximizing the sum-rate using zero-forcing precoding for downlink transmission with finite rate CSI feedback. The system assumes a BS with M antennas and K single antenna users.

$$y_k[n] = \mathbf{h}_k^H[n] \sum_{j \in \mathcal{S}} \mathbf{v}_j[n] s_j[n] + z_k[n], \ k \in \mathcal{S} \text{ and } n \geq 0, \quad (2)$$

where $z_k[n] \sim \mathcal{CN}(0,1)$ denotes a normalized additive noise. We assume that the time correlation variable a_k is perfectly known at the BS and each of the users, and we do not take into account feedback delay or feedback error.

Note that even before the nth feedback (where $n \geq 1$) is received, the BS has *a priori* knowledge about the channels of the selected users, $j \in \mathcal{S}$. When the *a priori* CSI at the nth time is denoted by $\hat{\mathbf{h}}_j[n-1]$ and a_j is the time correlation parameter, the present quantized vector $\hat{\mathbf{h}}_j[n]$ can be given by quantizing only the difference between the present channel vector and the *a priori* CSI (i.e., $\mathbf{d}_j[n] \triangleq \mathbf{h}_j[n] - a_j \hat{\mathbf{h}}_j[n-1]$). Then the BS and user quantizers can update their current channel information according to

$$\hat{\mathbf{h}}_j[n] = a_j \hat{\mathbf{h}}_j[n-1] + \hat{\mathbf{d}}_j[n], \quad (3)$$

where $\hat{\mathbf{d}}_j[n]$ is the quantized version of the difference vector $\mathbf{d}_j[n]$ and is also selected from each user's codebook composed of 2^B vectors. In contrast to prior work in [31] on predictive CSI quantization, the proposed feedback update algorithm uses only one codebook to quantize the difference vector and the initial CSI feedback vector. A detailed discussion is given in Section III. In this paper, a common codebook rotated by a user specific $M \times M$ random unitary matrix is used as each user's codebook. This codebook design guarantees the same average quantization distortion per user, and avoids ill-conditioning of the compiled matrix by creating different codebook for each user [47].

Each of the users feeds back the selected index to the BS, and each of the selected users and the BS update the present quantized vector based on (3). At each feedback update, the quantized vectors of the selected users are compiled into a matrix $\hat{\mathbf{H}}[n] = \left[\hat{\mathbf{h}}_{j_1}[n] \ \hat{\mathbf{h}}_{j_2}[n] \ \ldots \ \hat{\mathbf{h}}_{j_M}[n]\right]$. A zero-forcing precoder is considered at the BS using a precoding matrix given by

$$\mathbf{V}[n] = \hat{\mathbf{H}}[n] \left(\hat{\mathbf{H}}^H[n] \hat{\mathbf{H}}[n]\right)^{-1} \hat{\mathbf{C}}[n], \quad (4)$$

where $\hat{\mathbf{C}}[n]$ denotes a diagonal matrix corresponding to power allocation across the users. In this paper, we consider a power allocation which makes each precoding vector unit norm as in [7]. In other words, the mth diagonal term of $\hat{\mathbf{C}}[n]$ is given by $\hat{c}_{mm}[n] = \sqrt{\frac{1}{\left(\hat{\mathbf{H}}^H[n]\hat{\mathbf{H}}[n]\right)^{-1}_{mm}}}$, where $\mathbf{A}_{mm}$ denotes the (m,m)th element of $\mathbf{A}$.

III. A New Differential Feedback Scheme Inspired by Rate Distortion Theory

It is well known that quantizing only the CDI (i.e., $\mathbf{h}_j[n]/\|\mathbf{h}_j[n]\|$) is suitable for most applications of quantized feedback in single-user wireless systems with a multiple antenna transmitter and single antenna receiver. Multiuser MIMO systems, however, typically need more information about the users' channels to fully exploit multiuser diversity and maximize the sum-rate. In this section, we consider the quantization of a single channel vector. We first investigate the statistical properties of the difference vector $\mathbf{d}_j[n]$ and then propose a differential feedback scheme based on these properties. For simplicity, the user indices j and k will be removed based on the homogeneous characteristic of the users' channel distributions. The user indices are reemployed in Section IV to analyze the sum-rate and exploit multiuser diversity in the limited feedback zero-forcing precoding system.

A. Analysis of Quantization Error Bounds in Slowly Varying Channels

Using the quantization framework discussed in Section II, each scheduled user and the BS already have *a priori* information (i.e., $\hat{\mathbf{h}}[n-1]$) that can be used during the quantization $\mathbf{h}[n]$. With the channel evolution model in (1), the quantization of the difference vector $\mathbf{d}[n] = \mathbf{h}[n] - a\hat{\mathbf{h}}[n-1]$ is considered. Because the variance of the difference vector $\mathbf{d}[n]$ typically is smaller than the variance of the nth channel vector $\mathbf{h}[n]$ in slowly varying channels, we can expect improvement in the BS's CSI resolution by quantizing $\mathbf{d}[n]$. Differently from [31] which uses two codebooks for an initial channel vector and difference vectors and [30] which uses high resolution feedback for an initial channel vector, we use only one codebook under a Gaussian model for the previous quantization error vectors $\mathbf{e}[i] \triangleq \mathbf{h}[i] - \hat{\mathbf{h}}[i]$ for $0 \le i \le n-1$.

Lemma 1: Under a Gaussian assumption for the previous quantization error vectors $\mathbf{e}[i]$ for $0 \le i \le n-1$ (i.e., $\mathbf{e}[i] \sim \mathcal{CN}(\mathbf{0}, \frac{D_i}{M}\mathbf{I})$ with the quantization error distortion $D_i \triangleq E\left\|\mathbf{h}[i] - \hat{\mathbf{h}}[i]\right\|^2$), the nth time normalized quantization error distortion is bounded by

$$\frac{D_n}{M} \ge \left(1 - a^2\left(1 - 2^{-b}\right)\frac{1 - a^{2n}2^{-nb}}{1 - a^2 2^{-b}}\right)2^{-b}, \quad (5)$$

$$\frac{D_n}{M} \lessapprox \left(1 - a^2\left(1 - C2^{-b}\right)\frac{1 - \left(a^2 C 2^{-b}\right)^n}{1 - a^2 C 2^{-b}}\right)C2^{-b}, \quad (6)$$

with $b = B/M$ when each quantization bit rate per feedback update is fixed as B and $C = C_s + C_g$. Here, $C_g = \frac{3^M \Gamma^3\left(\frac{M+1}{3}\right)}{16M\Gamma(M)}$ and $C_s = (2M-1)G\left(\frac{2\pi^{(2M-1)/2}}{\Gamma((2M-1)/2)}\right)^{\frac{2}{2M-1}}$ are a gain (i.e., $\|\mathbf{h}[n]\|$) codebook constant and a shape (i.e., $\mathbf{h}[n]/\|\mathbf{h}[n]\|$) codebook constant based on high resolution quantization analysis [33], [48]. G is a quantizer dependent constant and the value for the best lattice quantizer is given in [49].

We prove this result in Appendix A.

Corollary 1: As time goes to infinity when $Ca^2 2^{-b} < 1$, the quantization error is bounded by

$$\left(\frac{1 - a^2}{1 - a^2 2^{-b}}\right)2^{-b} \le \frac{D_\infty}{M} \lessapprox \left(\frac{1 - a^2}{1 - a^2 C 2^{-b}}\right)C2^{-b}. \quad (7)$$

The quantization error of a conventional quantization method without using any *a priori* knowledge (i.e., $\hat{\mathbf{h}}[n-1]$) is still bounded by $2^{-b} \le \frac{D_\infty}{M}\left(= \frac{D_0}{M}\right) \lessapprox C2^{-b}$. In temporally correlated channels (i.e., $0 < a \le 1$), $\frac{1-a^2}{1-a^2C2^{-b}} < 1$ for $1 \le C < \frac{1}{2^{-b}}$. When a is close to 1 (i.e., the channel is highly correlated in time) both quantization distortion bounds converge to zero.

Remark 1: In fact, rate distortion theory for a Gaussian source gives a lower bound on the expected distortion, and the Gaussian lower bound becomes an equality when the quantization error is also Gaussian distributed. Even if the quantization error vector does not have exactly a Gaussian behavior, however, the quantization error of a closed-loop predictive quantizer, which repeatedly quantizes the difference between the current target and previously quantized vector and predicts the target [33], becomes Gaussian distributed even for a binary quantizer as time goes to infinity [50].

Remark 2: The nth time quantized channel is updated as in (3), and the current channel can be expressed by (22) in Appendix A. Under the same assumption as in *Lemma 1*, the conditional distribution of $\mathbf{h}[n]$ given $\hat{\mathbf{h}}[n-1]$ is $\mathbf{h}[n]\Big|\hat{\mathbf{h}}[n-1] \sim \mathcal{CN}(a\hat{\mathbf{h}}[n-1], \epsilon_n^2\mathbf{I})$, where $\epsilon_n^2 = \left(a^2\frac{D_{n-1}}{M} + 1 - a^2\right)$. Then $\mathbf{d}[n]\Big|\hat{\mathbf{h}}[n-1] \sim \mathcal{CN}\left(\mathbf{0}, \epsilon_n^2\mathbf{I}\right)$ and $\hat{\mathbf{d}}[n]$ can be expressed with ϵ_n and the quantization of a Gaussian vector with $\mathcal{CN}\left(\mathbf{0}, \mathbf{I}\right)$.

B. A New Differential Feedback Based on the Bound Analysis

As mentioned in *Remark 2*, $\hat{\mathbf{d}}[n]$ can be expressed using ϵ_n and a codeword from a B-bit zero mean and unit variance

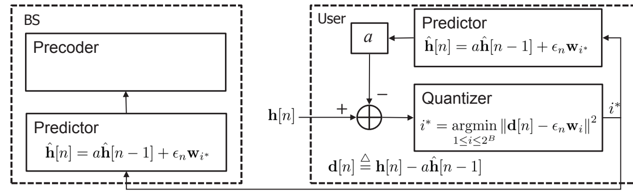

Fig. 2. Block diagram of the proposed differential feedback scheme at the nth channel realization using ϵ_n calculated by the time correlation and previous quantization distortion lower bound. The set-up uses a B-bit codebook $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{2^B}\}$.

Gaussian codebook $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_{2^B}\}$. The codebook $\mathcal{W}$ can be generated offline using techniques such as the generalized Lloyd algorithm [33]. Figure 2 depicts the proposed differential feedback, when a user and the BS have the Gaussian codebook and time correlation parameter. To avoid additional feedback about the variance of the difference vector, we can update ϵ_n based on the bounds in *Lemma 1*. For simplicity, we update ϵ_n using the lower bound in this paper with

$$\epsilon_n = 1 - a^2\left(1 - 2^{-b}\right)\frac{1 - a^{2n}2^{-nb}}{1 - a^2 2^{-b}}. \quad (8)$$

The algorithm can be summarized as follows.

1) Set $n = 0, \epsilon_0 = 1$, and $\hat{\mathbf{h}}[-1] = \mathbf{0}$. A user selects a codeword index i^* for feedback according to $i^* = \underset{1\le i\le 2^B}{\mathrm{argmin}} \|\mathbf{h}[0] - \epsilon_0\mathbf{w}_i\|^2$ because $\mathbf{d}[0] = \mathbf{h}[0]$. The user then feeds back the index i^* to the predictors in the user and the BS. Then $\hat{\mathbf{h}}[0] = \mathbf{w}_{i^*}$ at the output of the predictors.
2) Increase the feedback update time index as $n = n+1$. The user and the BS update ϵ_n based on (8).
3) The user computes the difference $\mathbf{d}[n]$ between the input and predictor output multiplied by a and selects $i^* = \underset{1\le i\le 2^B}{\mathrm{argmin}} \|\mathbf{d}[n] - \epsilon_n\mathbf{w}_i\|^2$. The index i^* is fed back to the predictors in the user and the BS.
4) The BS and the user predict $\hat{\mathbf{h}}[n] = a\hat{\mathbf{h}}[n-1] + \epsilon_n\mathbf{w}_{i^*}$ and go to 2).

Figure 3 shows a comparison of the quantization distortions between the proposed differential feedback scheme and previous feedback schemes, where basic feedback denotes the feedback of the codebook index closest to the current channel vector. Because the quantization distortion is given by the Euclidean distance normalized by the number of transmit antennas, the feedback method in [30] is compared with the proposed method. The normalized quantization distortion of the proposed quantizer is bounded by *Lemma 1*. In addition, both bounds become tight as time progresses. For comparison, the performances in Jakes' channel model are given. Note that our quantizer gives a lower quantization distortion than the approach in [30] in both channel models. This reduction in distortion is a direct result of the fact that our quantization update approach takes into account the effects of quantization error in prior updates *and* the channel's time correlation.

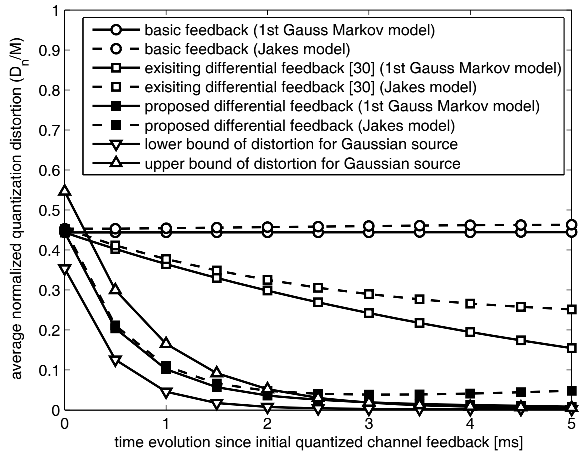

Fig. 3. Comparison of the quantization error distortions between the proposed differential feedback scheme and previous feedback schemes. Upper and lower bound distortions of the proposed differential feedback are given, where a 6-bit Lloyd-based Gaussian codebook is used. The simulation assumes $M = 4$ and uses Jakes' model with $15km/h$ mobility. A carrier frequency of $2GHz$ and a feedback update time of $0.5ms$ are used.

IV. A Multiuser MIMO System with the Proposed Differential Feedback

In a multiuser MIMO system employing Section III's proposed differential feedback, we analyze the sum-rate performance for the case of $K = M$ and propose a user selection algorithm for the case of $K > M$. The user indices j and k will be reemployed in this section, but we assume all users have the same temporal correlation statistics for simplicity.

By the orthogonality between $\hat{\mathbf{h}}_k[n]$ and $\mathbf{v}_j[n]$ for $j \neq k$ when a zero-forcing precoder is used, the received signal of the kth user given in (2) is rewritten as

$$y_k[n] = \mathbf{h}_k^H[n]\mathbf{v}_k[n]s_k[n] + \mathbf{e}_k^H[n]\sum_{j\neq k, j\in\mathcal{S}} \mathbf{v}_j[n]s_j[n] + z_k[n], \quad (9)$$

$$\text{for } \mathbf{e}_k[n] \triangleq \mathbf{h}_k[n] - \hat{\mathbf{h}}_k[n] \text{ and } k \in \mathcal{S}.$$

When equal power allocation is considered, the received SINR of the kth user is expressed by

$$\text{SINR}_k[n] = \frac{\rho\left|\mathbf{h}_k^H[n]\mathbf{v}_k[n]\right|^2}{1+\rho\sum_{j\neq k, j\in\mathcal{S}}\left|\mathbf{e}_k^H[n]\mathbf{v}_j[n]\right|^2},\ \rho \triangleq \frac{P}{M}, \quad (10)$$

where $E\left\|\mathbf{e}_k[n]\right\|^2 = D_n$. Then the sum-rate of the system with a zero-forcing precoder using quantized CSI is given by

$$R_Q[n] = ME\log_2\left(1+\frac{\rho\left|\mathbf{h}_k^H[n]\mathbf{v}_k[n]\right|^2}{1+\rho\sum_{j\neq k, j\in\mathcal{S}}\left|\mathbf{e}_k^H[n]\mathbf{v}_j[n]\right|^2}\right). \quad (11)$$

A. Throughput Analysis ($K = M$)

If full CSI at the transmitter (CSIT) is available, there is no interference after the zero-forcing precoding procedure. Given full CSIT, let $\mathbf{v}_{ZF,k}[n]$ be a zero-forcing precoding vector of the kth user. Then, the sum-rate is given by

$$R_F[n] = ME\log_2\left(1+\rho\left|\mathbf{h}_k^H[n]\mathbf{v}_{ZF,k}[n]\right|^2\right). \quad (12)$$

Proposition 1: Consider M users with the channel model given in (1) and downlink transmission defined by the zero-forcing precoding matrix given in (4). Assume that the error vector $\mathbf{e}_k[n]$ is uncorrelated with the corresponding quantized channel vector and i.i.d. elements. Then, an upper bound on the rate-loss per user of the B-bit finite feedback system compared to the full CSIT system is

$$\Delta R_n = \frac{R_F[n]-R_Q[n]}{M} \leq \log_2\left(1+\rho(M-1)\frac{D_n}{M}\right), \quad (13)$$

where the quantization distortion per transmit antenna $\frac{D_n}{M}$ is bounded by (5) and (6).

Proof: The rate-loss of the quantized system relative to the full CSIT system is bounded by

$$\begin{aligned}\Delta R_n \leq & E\log_2\left(1+\rho\left|\mathbf{h}_k^H[n]\mathbf{v}_{ZF,k}[n]\right|^2\right) \\ & - E\log_2\left(1+\rho\left|\mathbf{h}_k^H[n]\mathbf{v}_k[n]\right|^2\right) \\ & + E\log_2\left(1+\rho\sum_{j\neq k, j\in\mathcal{S}}\left|\mathbf{e}_k^H[n]\mathbf{v}_j[n]\right|^2\right) \quad (14) \\ = & E\log_2\left(1+\rho\sum_{j\neq k, j\in\mathcal{S}}\left|\mathbf{e}_k^H[n]\mathbf{v}_j[n]\right|^2\right). \quad (15)\end{aligned}$$

The inequality (14) follows from the fact that $\log_2\left(1+\frac{a}{1+b}\right) \geq \log_2(1+a) - \log_2(1+b)$ for positive values a and b. The equality (15) comes from the fact that $\left|\mathbf{h}_k^H[n]\mathbf{v}_k[n]\right|^2 \Big| \mathbf{v}_k[n] \sim \left|\mathbf{h}_k^H[n]\mathbf{v}_{ZF,k}[n]\right|^2 \Big| \mathbf{v}_{ZF,k}[n]$, where $\left|\mathbf{h}_k^H[n]\mathbf{v}_k[n]\right|^2 \Big| \mathbf{v}_k[n]$ and $\left|\mathbf{h}_k^H[n]\mathbf{v}_{ZF,k}[n]\right|^2 \Big| \mathbf{v}_{ZF,k}[n]$ are chi-square distributed [51] because of the unitary invariance of the i.i.d. Gaussian channel distribution of $\mathbf{h}_k[n]$.

The upper bound, (15), is bounded by Jensen's inequality as

$$\Delta R_n \leq \log_2\left(1+\rho(M-1)E\left|\mathbf{e}_k^H[n]\mathbf{v}_j[n]\right|^2\right) \text{ for } j \neq k.$$

Under the assumption that the error vector is uncorrelated with the corresponding quantized channel vector [2] and its elements are i.i.d., the expectation $E\left|\mathbf{e}_k^H[n]\mathbf{v}_j[n]\right|^2 = \frac{D_n}{M}E\left\|\mathbf{v}_j[n]\right\|^2 = \frac{D_n}{M}$. Therefore,

$$\Delta R_n \leq \log_2\left(1+\rho(M-1)\frac{D_n}{M}\right). \quad (16)$$

■

The averaged rate-loss per user, ΔR_n, obtained by simulation and its upper bounds are given in Fig. 4. The three upper bounds are described in (16), but each of them is calculated using a different quantization error distortion. One is calculated

[2]The error vector of the kth user is uncorrelated with quantized channel vectors, $\hat{\mathbf{h}}_j[n]$, $j \in \mathcal{S}$ and $j \neq k$ because of the independence of the quantized channels. Under the assumption, it is also uncorrelated with the corresponding quantized channel $\hat{\mathbf{h}}_k[n]$. Therefore, $\mathbf{e}_k[n]$ is uncorrelated with $\mathbf{v}_j[n]\mathbf{v}_j^H[n]$ given by $\mathbf{I} - \hat{\mathbf{H}}_{-j}[n]\left(\hat{\mathbf{H}}_{-j}^H[n]\hat{\mathbf{H}}_{-j}[n]\right)^{-1}\hat{\mathbf{H}}_{-j}[n]^H$, where $\hat{\mathbf{H}}_{-j}[n]$ denotes the matrix remaining after column $\mathbf{h}_j[n]$ is removed.

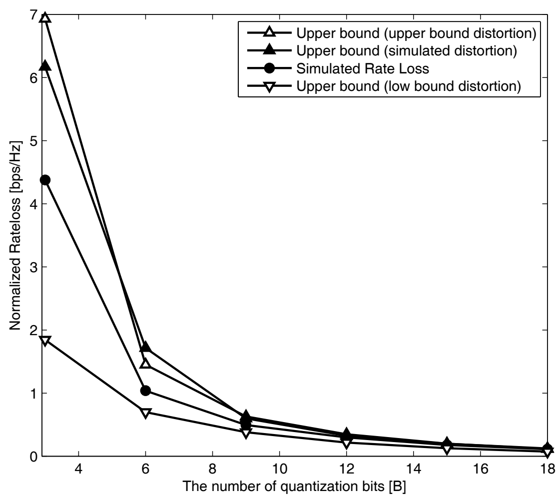

Fig. 4. Average rate-loss per user versus the number of quantization bits assuming Jakes' correlation with $15km/h$ mobility. Upper bounds of rate distortion are given according to different quantization distortion expressions. The simulation assumes $M = 4$, $K = 4$, and an SNR of 20dB. The rate-loss is given after 10 feedback updates i.e., after $5ms$ since an initial feedback update.

by simulating the expected distortion $\frac{D_n}{M}$. The other bounds are expressed by using the upper bound quantization distortion and lower bound quantization distortion respectively, given in *Lemma 1*. Figure 4 shows all the upper bounds grow tight as the number of quantization bits increases.

Remark 3: The procedure of the proof of *Proposition* 1 is mainly based on the proof of *Theorem 1* in [7]. However, we do not limit the codebook to RVQ in the proof. In this paper, we make use of the independence of the quantized channel vectors of the M users but not the isotropic property as in [7]. The averaged rate-loss per user is a function of the signal-to-noise ratio[3] (SNR) and the quantization distortion as given in [7], where the quantization distortion is a function of time in this paper.

Based on *Proposition* 1 and the distortion lower bound given in (7), we can quantify the appropriate amount of time correlation that guarantees a desired amount of improvement in the sum-rate as time goes to infinity. Compared to the basic feedback scheme that does not use the channels' temporal correlation, the rate-loss difference from the proposed differential feedback scheme is approximately given by

$$\Delta R_0 - \Delta R_\infty \approx \tag{17}$$

$$\log_2\left(1+\rho(M-1)2^{-b}\right) - \log_2\left(1+\frac{\rho(M-1)2^{-b}(1-a^2)}{1-a^2 2^{-b}}\right)$$

$$= \log_2\left(\frac{1}{\frac{1}{1+\rho(M-1)2^{-b}}\frac{a^2\left(1-2^{-b}\right)}{1-a^2 2^{-b}} + \frac{1-a^2}{1-a^2 2^{-b}}}\right). \tag{18}$$

Note that when $\rho(M-1)2^{-b} \gg 1$ (i.e., a high SNR case with nonnegligible quantization error), the normalized sum-rate using the differential feedback increases as much as the rate-loss difference given by $\log_2\left(\frac{1-a^2 2^{-b}}{1-a^2}\right)$. For example, if we want at least 1 bit per channel use rate increase per user when employing the differential feedback, the time correlation a should be larger than 0.779.

[3] In this paper, SNR denotes SNR per channel use defined by $\frac{P}{M}(=\rho)$, where a normalized additive noise is considered as given in (2).

B. Proposed User Selection Algorithm ($K > M$)

Using our feedback scheme when $K > M$ requires user selection. However, to employ user selection with a maximum sum-rate objective, SINR information is required at the BS. The BS does not have the exact SINR expressions that consider the interference between users because the BS receives only the quantized vector for each of the users as feedback in this paper. Because user selection techniques that use approximate SINR information or lower bounds of an expected SINR are known to provide excellent performance [39], [40], we suboptimally estimate the SINRs for a given compiled channel matrix expressed by quantized channels at the BS for our user selection scheme. Note that our user selection scheme requires no additional feedback (such as additional CQI).

Under the assumption that the error vector has zero mean i.i.d. Gaussian elements and is orthogonal to its corresponding quantized channel vector, the conditional expectation of SINR for the kth user is approximately bounded as (19). The inequality (19b) comes from the fact that $\mathbf{e}_k^H[n]\mathbf{V}[n]\mathbf{V}^H[n]\mathbf{e}_k[n] = \mathrm{Tr}\big(\mathbf{e}_k[n]\mathbf{e}_k^H[n]\mathbf{V}[n]\mathbf{V}^H[n]\big)$ and $\mathrm{Tr}(\mathbf{AB}) \le \lambda_{\mathrm{M}}(\mathbf{A})\,\mathrm{Tr}(\mathbf{B})$, $\lambda_{\mathrm{M}}(\mathbf{A})$ denotes the maximum eigenvalue of a matrix $\mathbf{A}$. Because of our assumptions about the error vector, $\|\mathbf{e}_k[n]\|^2$ is independent of $\left|\tilde{\mathbf{e}}_k^H[n]\mathbf{v}_k[n]\right|^2$, where $\tilde{\mathbf{e}}_k[n]$ is a normalized error vector given by $\frac{\mathbf{e}_k[n]}{\|\mathbf{e}_k[n]\|}$. In addition to the independence, the inequality (19c) comes from Jensen's inequality because SINR_k is a convex function of $\left|\tilde{\mathbf{e}}_k^H[n]\mathbf{v}_k[n]\right|^2$. The last inequality also comes from Jensen's inequality, but the approximation is given because SINR_k is not always a convex function of $\|\mathbf{e}_k[n]\|^2$. The exact inequality is true when

$$\rho\left|\hat{\mathbf{h}}_k^H[n]\mathbf{v}_k[n]\right|^2 \ge \frac{1}{M\lambda_{\mathrm{M}}\left(\mathbf{V}[n]\mathbf{V}^H[n]\right)-1}, \tag{20}$$

where the maximum eigenvalue of the precoding matrix is larger than 1. When the selected users are orthogonal to each other, $\lambda_{\mathrm{M}}\left(\mathbf{V}[n]\mathbf{V}^H[n]\right) = 1$, and then the condition (20) is $\rho\left|\hat{\mathbf{h}}_k^H[n]\mathbf{v}_k[n]\right|^2 \ge \frac{1}{M-1}$.

If we use one of the quantization distortion bounds as the expectation of the quantization error, the estimated SINR is a function of the quantized channels only. Using the estimated SINR, we propose a scheduler maximizing an approximate sum-rate given as

$$R_Q[n] \approx \sum_{k\in\mathcal{S}} \log_2\left(1+\frac{\rho\left|\hat{\mathbf{h}}_k^H[n]\mathbf{v}_k[n]\right|^2 + \rho\frac{D_n}{M}}{1+\rho D_n\left(\lambda_{\mathrm{M}}\left(\mathbf{V}[n]\mathbf{V}^H[n]\right)-\frac{1}{M}\right)}\right), \tag{21}$$

where the lower bound given in (5) is used for $\frac{D_n}{M}$ in this paper. We select a user set $\mathcal{S}$, consisting of M users among all $\binom{K}{M}$ user sets, maximizing (21).

In Fig. 5, we compare several user selection algorithms using two different forms of 12-bit random vector quantization (RVQ) user codebooks. One is given by normalized

$$E\left[\mathrm{SINR}_k[n]\middle|\hat{\mathbf{H}}[n]\right] = E\left[\frac{\rho\left|\hat{\mathbf{h}}_k^H[n]\mathbf{v}_k[n]\right|^2 + \rho\left|\mathbf{e}_k^H[n]\mathbf{v}_k[n]\right|^2}{1+\rho\left(\mathbf{e}_k^H[n]\mathbf{V}[n]\mathbf{V}^H[n]\mathbf{e}_k[n] - \left|\mathbf{e}_k^H[n]\mathbf{v}_k[n]\right|^2\right)}\middle|\hat{\mathbf{H}}[n]\right] \tag{19a}$$

$$\geq E_{\|\mathbf{e}_k[n]\|^2, \left|\tilde{\mathbf{e}}_k^H[n]\mathbf{v}_k[n]\right|^2}\left[\frac{\rho\left|\hat{\mathbf{h}}_k^H[n]\mathbf{v}_k[n]\right|^2 + \rho\|\mathbf{e}_k[n]\|^2\left|\tilde{\mathbf{e}}_k^H[n]\mathbf{v}_k[n]\right|^2}{1+\rho\|\mathbf{e}_k[n]\|^2\left(\lambda_{\mathrm{M}}\left(\mathbf{V}[n]\mathbf{V}^H[n]\right) - \left|\tilde{\mathbf{e}}_k^H[n]\mathbf{v}_k[n]\right|^2\right)}\middle|\hat{\mathbf{H}}[n]\right] \tag{19b}$$

$$\geq E_{\|\mathbf{e}_k[n]\|^2}\left[\frac{\rho\left|\hat{\mathbf{h}}_k^H[n]\mathbf{v}_k[n]\right|^2 + \rho\frac{\|\mathbf{e}_k[n]\|^2}{M}}{1+\rho\|\mathbf{e}_k[n]\|^2\left(\lambda_{\mathrm{M}}\left(\mathbf{V}[n]\mathbf{V}^H[n]\right) - \frac{1}{M}\right)}\middle|\hat{\mathbf{H}}[n]\right] \tag{19c}$$

$$\gtrapprox \frac{\rho\left|\hat{\mathbf{h}}_k^H[n]\mathbf{v}_k[n]\right|^2 + \rho\frac{D_n}{M}}{1+\rho D_n\left(\lambda_{\mathrm{M}}\left(\mathbf{V}[n]\mathbf{V}^H[n]\right) - \frac{1}{M}\right)}, \tag{19d}$$

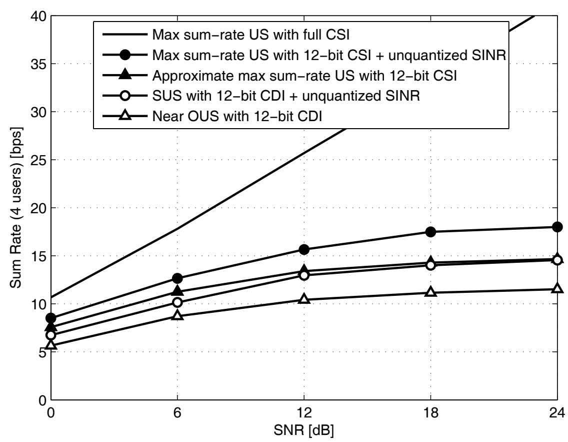

Fig. 5. The average sum-rates are shown versus SNR when several user selection algorithms are considered. In the simulation, $M = 4, K = 50$ and 12-bit RVQ codebook is used.

Gaussian random vectors, and the other is given by Gaussian random vectors themselves. An orthogonal user selection (OUS) algorithm is performed because the scheduler can select users orthogonal to other selected users only with CDI. For simplicity, a near OUS selects a user set $\mathcal{S}$ maximizing the minimum eigenvalue of the precoding matrix among all $\binom{K}{M}$ user sets. In addition to the 12-bit CDI, the SUS algorithm considers additional SINR feedback and can increase the sum-rate [34]. Compared to the near OUS and SUS algorithms, the proposed scheduler uses the quantization of the channel vector itself as the CSI feedback. Because of the different codebooks employed, the precoder and SINR expressions of the proposed scheduler are different from those used in the near OUS and SUS algorithms. For comparison, the sum-rate of a user selection scheme maximizing the sum-rate given by the exact SINR in (10) is shown. By using the the lower bound of the estimated SINR given in (19d) using $\frac{D_0}{M}$ (because the BS does not have *a priori* knowledge at the user selection stage as mentioned in Section II), the proposed user selection does not require additional SINR feedback at the cost of sum-rate performance. As shown in Fig. 5, however, we can increase sum-rate performance compared to the near OUS algorithm with only CDI at the same feedback rate. In addition, the proposed user selection scheme shows better sum-rate performance than the SUS algorithm in a moderate SNR region, even though the SUS algorithm requires additional SINR feedback information.

Remark 4: In this section, we dealt with the case when all users have the same time correlation parameter for simplicity. If the time correlation parameter is different for each user, the quantization error distortion is a function of the time correlation parameter and is different for each user. Then the normalized rate-loss bound given in (16) is replaced with $\Delta R_n \leq \frac{1}{M}\sum_{k=1}^{M}\log_2\left(1+\rho(M-1)\frac{D_n^{(k)}}{M}\right)$, where $\frac{D_n^{(k)}}{M}$ is a function of a_k. Also, the approximate sum-rate given in (21) is replaced with $R_Q[n] \approx \sum_{k\in\mathcal{S}}\log_2\left(1+\frac{\rho\left|\hat{\mathbf{h}}_k^H[n]\mathbf{v}_k[n]\right|^2+\rho\frac{D_n^{(k)}}{M}}{1+\rho D_n^{(k)}\left(\lambda_{\mathrm{M}}(\mathbf{V}[n]\mathbf{V}^H[n])-\frac{1}{M}\right)}\right)$.

V. Simulation Results

Using Monte Carlo simulations, we compare the sum-rates of the proposed differential feedback with basic feedback that does not consider time correlation and previous schemes utilizing the channels' time correlation [30]–[32]. For time varying channels, Jakes' channel model is used, where $15km/h$ and $50km/h$ channels are used at $2GHz$ center frequency. Each time slot is $0.5ms$ for the feedback update. The sum-rates of the proposed differential feedback are given versus the SNR or the feedback update time, where SNR indicates SNR per channel use. In addition, we show the performance of the proposed user selection algorithm with the proposed differential feedback. In our simulations, a six bit Gaussian codebook is designed using the Lloyd algorithm for the basic feedback, previous predictive quantizer [30], and proposed differential feedback. As mentioned, the gain of the difference channel vector in the proposed algorithm ϵ_n is updated by (8). For the algorithm in [31], the same codebook is used for the channel codebook, and the additional codebooks for difference channel vector quantization are learned by a Lloyd algorithm for the channels with different mobilities. The 6 bit Grassmannian codebook is used for the algorithm in [32].

Figure 6 shows the average sum-rates for a system with four users versus the feedback update time when the SNR is fixed at $19dB$. Since the quantization error of the differential feedback is bounded as the feedback update time progresses by (7), the

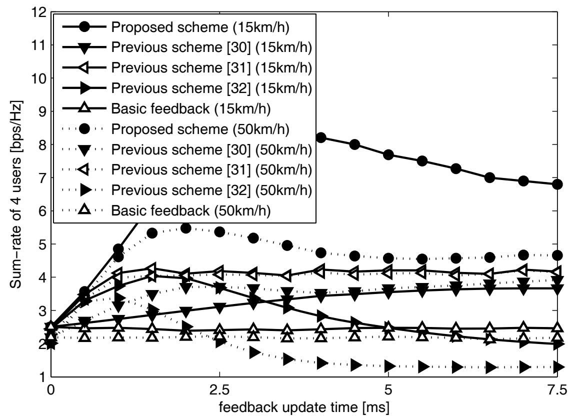

Fig. 6. The average sum-rates are shown versus the feedback update time according to two mobilities, $15km/h$ and $50km/h$. As the number of iteration updates increases, the average sum-rates of the proposed differential feedback saturate. The simulation assumes $M = 4, K = 4$, and $19dB$ SNR.

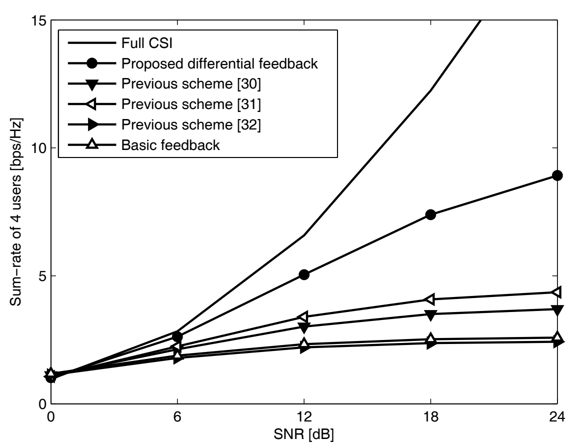

Fig. 7. Average sum-rates plotted versus SNR after $5ms$, where $M = 4$ and $K = 4$.

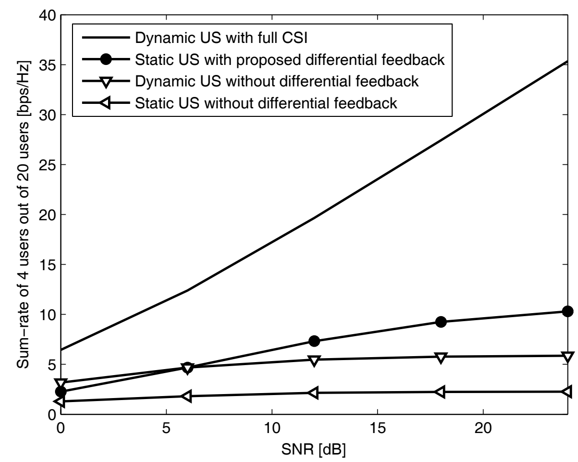

Fig. 8. Average sum-rates plotted versus SNR. In the low SNR region, the effect on the sum-rate of multiuser diversity due to dynamic user selection is dominant, and the effect of quantization distortion reduction with differential feedback scheme is dominant in most other SNR regimes. The simulation assumes $M = 4, K = 20$ and the differential feedback results are given after $5ms$ since user selection stage.

sum-rate will asymptotically saturate. The proposed algorithm shows a sum-rate increase of about $5.5bps/Hz$ in a $15km/h$ channel and $2.5bps/Hz$ in a $50km/h$ over the basic feedback scheme at the $5ms$ feedback time. Although the scheme in [31] uses an additional codebook for the difference channel vector, the feedback update gain is reduced after the first few updates because the first order open-loop predictive quantizer is used in this simulation. As seen in Fig. 3, the reduction in quantization error compared with the scheme in [30] yields a sum-rate increase in Fig. 6. For reference, the result of the scheme in [32] is given, where its feedback rate is less than other algorithms after $1ms$.

When the feedback update time is fixed at $5ms$, the average sum-rates versus the SNR are plotted in Fig. 7. As shown in (16), the rate-loss is a function of the product between the SNR and quantization error. Therefore, the average sum-rate difference between full CSI precoding and 6-bit precoding grows as the SNR increases. Similarly, the sum-rate difference between the proposed differential feedback scheme and the conventional scheme increases in the high SNR regime.

In our algorithm, M users (maximizing the approximate sum-rate given in (21)) are selected using the first feedback information sent from all users. Then the codebook can be updated by using differential feedback. We call this static user selection with differential feedback in this paper, compared to dynamic user selection which selects M users out of K at each feedback time. The average sum-rates in the case of $K > M$ versus SNR are shown in Fig. 8. In dynamic user selection, the average sum-rates are increased by multiuser diversity regardless of the use of differential feedback. Interestingly, the static user selection with differential feedback shows a sum-rate increase of about $4bps/Hz$ over dynamic user selection without differential feedback. We can see that the quantization distortion reduction obtained by using differential feedback (at the cost of a reduction in multiuser diversity incurred by static user selection) can yield a more substantial sum-rate increase than dynamic user selection without differential feedback. The result is in line with the result in [44] which shows accurate channel information from a small number of users is more important than multiuser diversity arising from coarse channel information from a larger number of users in multiuser downlink systems with limited feedback.

Note that in our simulations we assumed that the time correlation parameter a was known to all of the users and the BS. In this paper, our main focus is to develop a limited rate differential feedback to improve the channel quantizer performance in slowly varying channels. The improvement in the quality of the channel quantizer is possible by updating the previously quantized channel based on the quantization error analysis and differential feedback. The analysis is performed given full knowledge of the a. However, the effect of the estimation error of the correlation on the performance is an interesting point to study. To demonstrate the effect of estimation error, simulations were done using an estimated temporal channel correlation parameter, where the correlation is estimated by averaging the normalized auto-correlation, for the proposed algorithm and the algorithm in [30]. As shown in

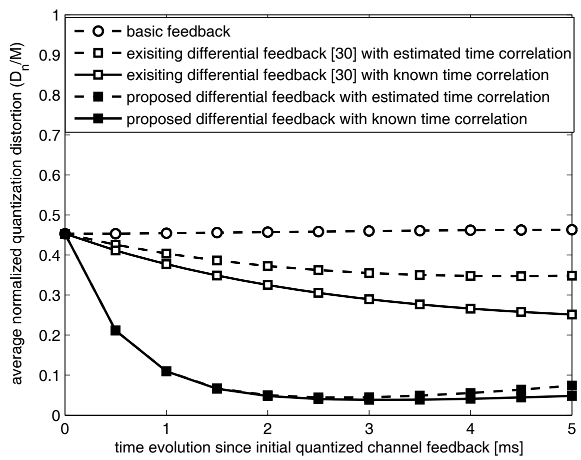

Fig. 9. Comparison of the quantization error distortions between the proposed differential feedback scheme and previous feedback schemes with estimated time correlation a, assuming Jakes correlation with $15km/h$ mobility. In the simulation, a 6-bit Lloyd-based Gaussian codebook is used and $M = 4$.

Fig. 9, the proposed algorithm is less sensitive to a feedback or estimation error than the previous algorithm [30] because it is already biased by quantization error.

VI. Conclusions

In this paper, we i) proposed a differential feedback scheme utilizing the presence of temporal channel correlation and previous quantization distortion and ii) developed a user scheduling algorithm to maximize the approximate sum-rate in the multiuser MIMO downlink with a zero-forcing precoder. When a zero mean and unit variance Gaussian codebook and time correlation parameter are given at both the BS and each of the users, the proposed differential feedback scheme updates of the previously fed back CSI using a technique inspired by rate distortion theory and a Gaussian quantization error assumption. In a multiuser MIMO system with a zero-forcing precoder using the proposed differential feedback scheme, we analyzed the impact of the feedback amount on sum-rate performance when $K = M$, and an estimated SINR using the quantized CSI and a quantization error lower bound to exploit multiuser diversity when $K > M$. Due to our developed SINR lower bound given in (19d), our algorithm can provide multiuser diversity gain without additional SINR feedback. Simulations showed that the proposed feedback scheme increases the sum-rate as the feedback is updated and outperforms previously proposed differential feedback schemes [30]–[32]. The proposed user selection algorithm is better than the SUS algorithm for moderate SNRs with less feedback information. In addition, the quantization distortion reduction by differential feedback can increase the average sum-rate more than user selection at each time without exploiting temporal channel correlation in slowly varying channels.

Appendix A
Proof of Lemma 1

Proof: We assume the channel follows a Gauss-Markov process as discussed in (1). For a given previous quantized channel $\hat{\mathbf{h}}[n-1]$ and quantization error vector $\mathbf{e}[n-1]$, the nth channel is expressed as

$$\mathbf{h}[n] = a\hat{\mathbf{h}}[n-1] + a\mathbf{e}[n-1] + \sqrt{1-a^2}\boldsymbol{\delta}[n], \qquad (22)$$

where $\mathbf{e}[n-1] \sim \mathcal{CN}(\mathbf{0}, \frac{D_{n-1}}{M}\mathbf{I})$. For the difference vector $\mathbf{d}[n] \triangleq \mathbf{h}[n] - a\hat{\mathbf{h}}[n-1]$, $\mathbf{d}[n]\Big|\hat{\mathbf{h}}[n-1] \sim \mathcal{CN}(\mathbf{0}, \epsilon_n^2\mathbf{I})$, where $\epsilon_n^2 = \frac{E\left\|a\mathbf{e}[n-1]+\sqrt{1-a^2}\boldsymbol{\delta}[n]\right\|^2}{M} = a^2\frac{D_{n-1}}{M} + 1 - a^2$.

Rate distortion theory shows that a lower bound on the b-bit quantization distortion per element with distribution of $\mathcal{CN}(0,\sigma^2)$ is given as $\sigma^2 2^{-b}$ [52]. The b-bit quantization error of the nth difference vector with variance ϵ_n is bounded as

$$\frac{D_n}{M} \geq \epsilon_n^2 2^{-b} = \left(a^2\frac{D_{n-1}}{M} + 1 - a^2\right)2^{-b} \qquad (23)$$

$$\geq \left(a^2 2^{-b}\right)^n 2^{-b} + \left(1-a^2\right)2^{-b}\sum_{i=0}^{n-1}\left(a^2 2^{-b}\right)^i$$

$$= \left(1 - a^2\left(1-2^{-b}\right)\frac{1-a^{2n}2^{-nb}}{1-a^2 2^{-b}}\right)2^{-b}. \qquad (24)$$

Based on the Gaussian assumption of previously quantized error vectors, rate distortion theory gives $\frac{D_0}{M} = 2^{-b}$, $\frac{D_1}{M} = \left(a^2\frac{D_0}{M} + 1 - a^2\right)2^{-b}, \ldots, \frac{D_{n-1}}{M} = \left(a^2\frac{D_{n-2}}{M} + 1 - a^2\right)2^{-b}$. By expanding the recurrence, we can obtain (24).

The upper bound follows from the fact that a shape-gain separate quantization method produces larger quantization error than shape-gain joint quantization [33]. Here we consider Lloyd-based entire unnormalized vector quantization as one of shape-gain joint quantization method. Let $\hat{g}$ and $\hat{\mathbf{f}}$ be a gain and shape quantization of $\mathbf{h} = g\mathbf{f} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$, respectively. Let the upper bound distortion of shape-gain joint quantization be $\frac{D_{gs}}{M}$. From [48], the quantization distortion per element of the B-bit shape-gain independent quantizer of a random vector with $\mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$ is given by

$$\frac{D_{gs}}{M} \triangleq \frac{1}{M}E\left\|\mathbf{h} - \hat{g}\hat{\mathbf{f}}\right\|^2 \qquad (25)$$

$$= \frac{E|g-\hat{g}|^2}{M} + \left(\sigma^2 - \frac{E|g-\hat{g}|^2}{M}\right)E\left\|\mathbf{f} - \hat{\mathbf{f}}\right\|^2 \qquad (26)$$

$$\leq \frac{E|g-\hat{g}|^2}{M} + \sigma^2 E\left\|\mathbf{f} - \hat{\mathbf{f}}\right\|^2 \qquad (27)$$

$$\approx \sigma^2\left(C_s 2^{-2MR_s/(2M-1)} + C_g 2^{-2MR_g}\right). \qquad (28)$$

The equality (26) comes from the independence of g and $\hat{g}$ from $\mathbf{f}$ and the centroid condition of the gain quantizer. The inequality (27) is tight when the error variance of a gain quantizer is small (i.e., a high-resolution quantization case). In (28), a lattice quantizer is considered as a shape quantizer and high resolution analysis of shape-gain quantization of a real vector in [48] is extended to a complex vector quantization (i.e., a $2M$ dimensional real vector with distribution $\mathcal{N}(0, \frac{\sigma^2}{2})$ is considered instead of an M dimensional complex vector with distribution $\mathcal{CN}(0,\sigma^2)$). Then shape quantization distortion and gain quantization distortion can be considered as $C_s 2^{-2MR_s/(2M-1)}$ and $C_g 2^{-2MR_g}$ respectively, based on a high-resolution quantization approach [48], where R_s and R_g denote the normalized number of quantization bits for shape and gain, respectively. The constant C_g is given by Bennett's integral [33] for the generalized Rayleigh distribution [48] corresponding to a $2M$ dimensional random vector with independent elements with $\mathcal{N}(0, \frac{\sigma^2}{2})$ as

$$C_g = \frac{1}{\sigma^2}\frac{1}{12M}\left(\int_0^\infty |f_g(r)|^{1/3}dr\right)^3 = \frac{3^M\Gamma^3\left(\frac{M+1}{3}\right)}{16M\Gamma(M)}, \quad (29)$$
$$\text{for } f_g(r) = \frac{2r^{2M-1}\exp(\frac{-r^2}{\sigma^2})}{\Gamma(M)\sigma^{2M}}.$$

In addition, the constant C_s of a shape quantizer for a $2M$ dimensional unit norm random vector (i.e., corresponding to a lattice quantizer of a $2M-1$ dimensional uniform source [48]) is given by $C_s = (2M-1)G\left(\frac{2\pi^{(2M-1)/2}}{\Gamma((2M-1)/2)}\right)^{\frac{2}{2M-1}}$, where G is a constant of a lattice quantizer and given in [49] for the best known lattice quantizers of a uniform source. Then, the optimal quantization bit allocation [48] of R_s and R_g, minimizing total distortion while satisfying $R_s + R_g = b$, is given by

$$R_s = \frac{2M-1}{2M}b + \frac{2M-1}{(2M)^2}\log_2\left(\frac{C_s}{C_g}\frac{1}{2M-1}\right) \quad (30)$$
$$R_g = \frac{1}{2M}b - \frac{2M-1}{(2M)^2}\log_2\left(\frac{C_s}{C_g}\frac{1}{2M-1}\right). \quad (31)$$

The optimal bit allocation in the high-resolution case can be approximated as $R_s = \frac{2M-1}{2M}b$ and $R_g = \frac{1}{2M}b$ for simplicity. This means that the shape codebook represents unit norm vectors with $(2M-1)$ degree of freedom by using $2^{(2M-1)b}$ codewords and the gain codebook expresses a scalar value with 2^b codewords. Therefore, each distortion decays at the same rate of 2^{-b} asymptotically. Finally the quantization distortion is upper bounded by $\frac{D_{gs}}{M} \lessapprox \sigma^2(C_s + C_g)2^{-b}$. Thus, $\frac{D_0}{M} \lessapprox C2^{-b}$, $\frac{D_1}{M} \lessapprox \left(a^2\frac{D_0}{M} + 1 - a^2\right)C2^{-b}$, $\cdots$, $\frac{D_{n-1}}{M} \lessapprox \left(a^2\frac{D_{n-2}}{M} + 1 - a^2\right)C2^{-b}$. By the same recurrence procedure used to derive the lower bound, the approximate upper bound is given as

$$\frac{D_n}{M} \lessapprox \left(1 - a^2\left(1 - C2^{-b}\right)\frac{1-\left(a^2C2^{-b}\right)^n}{1 - a^2C2^{-b}}\right)C2^{-b}. \quad (32)$$

■

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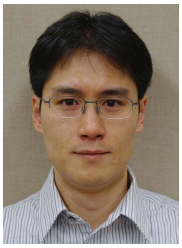
Kyeongyeon Kim (S'06 - M'10) received the B.S., M.S. and Ph.D. degrees in electrical engineering from Yonsei University, Seoul, Korea, in 2001, 2003 and 2007, respectively. As a postdoctoral fellow, she was with Purdue University from 2007 to 2008 and with University of Illinois at Urbana-Champaign from 2008 to 2010. Since September 2010, she has been with Samsung Advanced Institute of Technology (SAIT), Samsung Electronics, Korea. Her research interests are in communication and signal processing area, with particular emphasis on array signal processing, analysis and design of communication systems, iterative detection and decoding, and software defined radio (SDR).



Taejoon Kim (S'08) received the B.S. degree (with highest honors) in electrical engineering from Sogang University, Seoul, Korea, in 2002, and M.S. degree in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 2004. From 2004 to 2006, he was with Electronics and Telecommunications Research Institute (ETRI), Daejeon, Korea. Since 2007, he has worked towards the Ph.D. degree at Purdue University and received the Ph.D. degree in electrical engineering in 2011. He was a Summer Intern in the Samsung R&D Center, Richardson, TX, in 2008 and in DSPS R&D Center, Texas Instrument, Dallas, TX, in 2010, respectively. He is currently with Nokia Research Center (NRC), Berkeley, CA.

David J. Love (S'98 - M'05 - SM'09) received the B.S. (with highest honors), M.S.E., and Ph.D. degrees in electrical engineering from the University of Texas at Austin in 2000, 2002, and 2004, respectively. During the summers of 2000 and 2002, he was with Texas Instruments, Dallas, TX. Since August 2004, he has been with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN, where he is now an Associate Professor. Dr. Love currently serves as an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING. He has previously served as an Associate Editor for IEEE TRANSACTIONS ON COMMUNICATIONS and as a guest editor for special issues of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS and the *EURASIP Journal on Wireless Communications and Networking*. His research interests are in the design and analysis of communication systems and MIMO array processing.

Dr. Love has been inducted into Tau Beta Pi and Eta Kappa Nu. Along with co-authors, he was awarded the 2009 IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY Jack Neubauer Memorial Award for the best systems paper published in the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY in that year. He was the recipient of the Fall 2010 Purdue HKN Outstanding Teacher Award. In 2003, Dr. Love was awarded the IEEE Vehicular Technology Society Daniel Noble Fellowship.

Il Han Kim (S'07 - M'09) received the B.S. and M.S. degrees in electrical engineering from Korea Advanced Institute of Science and Technology in 2002 and 2004, respectively, and Ph.D. degree in electrical engineering from Purdue University in 2008.

During 2004 and 2005, he was with the Electronics and Telecommunications Research Institute, Daejeon, Republic of Korea. During the summers of 2007 and 2008, he was with the DSPS R&D Center, Texas Instruments Incorporated, Dallas, TX. Since January 2009, he has been with the Systems and Applications R&D Center, Texas Instruments Incorporated. His research interests include the analysis of communication systems.